WNE Linear Algebra Resit Exam
Series B

2 March 2019

Please use separate sheets for different problems. Please provide the following data on each sheet

- **name, surname and your student number,**
- **number of your group,**
- **number of the corresponding problem and the series.**

Problem 1.
Let $V = \text{lin}((1, 1, -5, -1), (-1, 3, -11, 13), (-3, 1, -1, 15))$ be a subspace of $\mathbb{R}^4$.

a) find a basis $\mathcal{A}$ of V and the dimension of V,
b) find all $t \in \mathbb{R}$ such that $(1, 2, -9, t) \in V$.

Problem 2.
Let $V \subset \mathbb{R}^4$ be a subspace given by the homogeneous system of linear equations

$$\begin{cases} x_1 - 4x_2 - x_3 - 2x_4 = 0 \\ x_1 - 4x_2 - 3x_3 + 2x_4 = 0 \\ 3x_1 - 12x_2 - 5x_3 - 2x_4 = 0 \end{cases}$$

a) which of the following sequences of vectors are bases of V?
   i) $((1, -4, -1, -2))$,
   ii) $((1, -4, -1, -2), (1, -4, -3, 2))$,
   iii) $((4, 1, 0, 0), (4, 0, 2, 1))$,
   iv) $((4, 1, 0, 0), (0, -1, 2, 1))$,
   v) $((-8, 0, -4, -2), (4, 0, 2, 1))$.
   Give reasons for your answers.
b) find coordinates of $w = (-8, -3, 2, 1)$ relative to one of the basis from the part a).

Problem 3.
Let

$$A = \begin{bmatrix} -1 & 1 \\ -6 & 4 \end{bmatrix}.$$

a) does there exists matrix $C \in M(2 \times 2; \mathbb{R})$ such that

$$C^{-1}AC = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$$

   for some $a, b \in \mathbb{R}$ such that $a > b$? If it does, give an example of such matrix C.
b) compute A^{200}.

Problem 4.
Let $\mathcal{A} = ((1, 1), (1, 2))$, $\mathcal{B} = ((1, 0), (1, 1))$ be ordered bases of $\mathbb{R}^2$. Let $\varphi\colon \mathbb{R}^2 \to \mathbb{R}^2$

be a linear transformations given by the matrix

$$M(\varphi)_{\mathcal{A}}^{\mathcal{B}} = \begin{bmatrix} 2 & 2 \\ -1 & 1 \end{bmatrix}.$$

Let $\psi\colon \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformations given by the matrix

$$M(\psi)_{\mathcal{A}}^{st} = \begin{bmatrix} 2 & 0 \\ 1 & -1 \\ 0 & 1 \end{bmatrix}.$$

a) find the formula of φ,
b) find the matrix $M(\psi \circ \varphi)_{\mathcal{B}}^{st}$.

Problem 5.
Let

$$A = \begin{bmatrix} 1 & 1 & 2 \\ -4 & t & -6 \\ 2 & 3 & 5 \end{bmatrix}.$$

a) for which $t \in \mathbb{R}$ is matrix $A^{\mathsf{T}} A^3$ invertible?
b) for which $t \in \mathbb{R}$ is the entry in the first row and the third column of A^{-1} equal to -3?

Problem 6.
Let $V = \operatorname{lin}((1,1,2),(3,5,8),(2,-1,1))$ be a subspace of $\mathbb{R}^3$.
a) find an orthonormal basis of $V^{\perp}$,
b) compute the orthogonal projection of $w = (0,0,1)$ onto V.

Problem 7.
Let M be the affine plane in $\mathbb{R}^3$ passing through the point $Q = (2,1,4)$ which is parallel to the subspace

$$V = \{(x_1,x_2,x_3) \in \mathbb{R}^3 \mid x_1 + 2x_3 = 0\}.$$

a) find a parametrization of the line passing through point $P = (1,1,2)$ perpendicular to M,
b) find an equation describing M and compute the affine orthogonal projection of the point P onto M.

Problem 8.
Consider the following linear programming problem $-2x_3 + 2x_4 \to \min$ in the standard form with constraints

$$\begin{cases} x_1 \; + \qquad\qquad\qquad + \; x_4 = 6 \\ \qquad\quad x_2 \; + \qquad\quad + \; 2x_4 = 2 \\ \qquad\qquad\qquad x_3 \; - \; 2x_4 = 1 \end{cases} \quad \text{and } x_i \geqslant 0 \text{ for } i = 1, \dots, 4$$

a) which of the sets $\mathcal{B}_1 = \{1,2,4\}, \mathcal{B}_2 = \{1,2,3\}$ is basic feasible? Write the corresponding feasible solution.
b) solve the linear programming problem using simplex method.